Structural Response of a Standalone FPSO by Swell Wave in Offshore Nigeria

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Abstract--- Deep water exploration has significantly increased the use of FPSOs. The reason been that FPSO provides an economic and flexible approach to exploration of oil and gas. However, these moored offshore structures are subjected to wave forces especially the swell effects in offshore Nigeria. Using mathematical and graphical computational tool, computations have been made on how the structure will respond on swell experienced in offshore Nigeria. The conditions considered in this paper include swell actions caused by the following swell waves, beam and head swell which is directly responsible for the surge, sway and heave motions.

Index Terms--- swell wave, structural response, FPSO, surge, sway, heave, standalone

1 Introduction

When waves come in contact with solid matter, energy is transferred to such objects, making such bodies to produce a response which can either be one or multi direction. In respect to FPSO responds to wave it results in the combination of surge, sway, heave, pitch, yaw and roll. Which is termed the six (6) degree of freedom [1]. The response prediction of ocean structures in a seaway is practised in design and installation of offshore structures through the application of the linear superposition principle in stochastic processes which was first introduced in the industry.

The present interest in swell wave impact analysis comes from the active deep water development which is presently taking place in offshore West Africa. In 2004 Olagnon et al described the West African offshore environment which ought to be generally mild because of less wind impact as persistent and can reach fairly high amplitudes with very low periods. [2] Which results from it receiving swell wave from the storms of Southern Ocean.

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The FPSO model used in this paper possesses an overall length of 280.4m, Beam = 53.61m, deadweight of 155312T, gross tonnage of 80023T.

2 Methodology

The mean wave (drift) force coefficients are generated from the wave excitation in the hydrodynamics analysis using the theories stated in this work. The theory was modified such that it considered Newman (1997) [3][6] and Chakarbarti (1987) [4][7] which suggested the best wave model for design and operation of ship-shaped structures in West Africa region. However, the proposed coefficient for current by Jensen (2004) [5][8] is not applied to my calculations since this work considers only swell wave. The work also established a relationship between wavelength and response amplitude operator.

Note, the scope of this paper is limited to the hull response as a result analysis on the mooring system have been neglected.

2.1 Frequency of Encounter.

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When a vessel moored oscillate at a particular frequency **ω** and a wave direction $\boldsymbol{\mu}$, the frequency at which it encounters the waves (ω_e) becomes important even though the significant wave height may be smaller than that of a fully developed sea, since the modal frequency is higher the heave motions tend to have higher natural frequencies.

The relationship between the frequency of encounter and the wave frequency becomes:

$$\omega_e = \omega - kV\cos\boldsymbol{\mu}$$

Note that $\boldsymbol{\mu} = 0$ for following waves.

2.2 Forces and moment responsible for FPSO response on swell

The steady drift forces and moments for an FPSO subjected to arbitrary waves neglecting current coefficient is expressed as:

$$F_i = \frac{1}{2}\rho g.\xi^2 \int_{L1}^{L2} sin^2(\Theta + \beta) ni.dl \quad 1$$

So the steady ***Surge*** and ***Sway*** drift forces and ***heave*** drift moment for a ship shaped structure can be expressed as:

Surge: $F1 = \frac{\rho g \xi^2}{2} \int sin^2(\Theta + \beta).sin\Theta.dl \quad 2$

Sway: $F2 = \frac{\rho g \xi^2}{2} \int sin^2(\Theta + \beta).cos\Theta.dl \quad 3$

Heave: $F3 = \frac{\rho g \xi^2}{2} \int sin^2(\Theta + \beta).(Xcos\Theta - Ysin\Theta).dl \quad 4$

2.3 The linear response

Whenever a force is induced into a body, the body tends to move or remained at rest depending on the magnitude of the exciting force. The structural response of an FPSO on swell wave is simply how far the structure displaced, when acted upon by a certain magnitude of force. This response is always in the same direction of the induced wave. [6][3] The response of the FPSO is determined in this paper by first defining its initial points defined on the hull of the FPSO, in this case the centre of gravity. Such that the responses becomes the deviation from the centre of gravity.

The FPSO motions in the steadily translating *O(x,y,z)* systems are defined by the three (3) translations of the vessel's centre of gravity(CG) in the direction of the x-, y- and z-axes and three (3) rotations about them as given in figure 1.

$$\text{Surge} = x = X_a\cos(\omega_e + \mathcal{E}_x\xi)$$

$$\text{Sway} = y = Y_a\cos(\omega_e + \mathcal{E}_y\xi)$$

$$\text{Heave} = \boldsymbol{\alpha} = \boldsymbol{\alpha}_a\cos(\omega_e + \mathcal{E}_{\boldsymbol{\alpha}}\xi)$$

<u>Note</u> that each of the ξ values is at a different phase angle.

Where;

$\mathcal{E}$ = Phase gap.

ω_e = Encountered frequency

X_a = Surge Amplitude

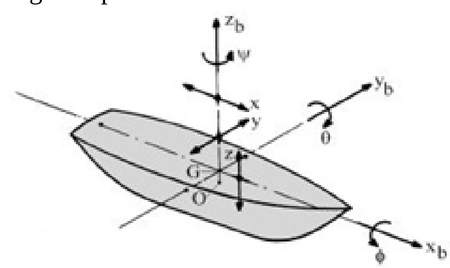

Figure 1: Defination of FPSO motions in six degree of freedom.

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$\omega_e = k(c - v\cos\boldsymbol{\mu})$

$90^0 \le \boldsymbol{\mu} \ge 180^0$ for head wave.

$0^0 \le \boldsymbol{\mu} \ge 90^0$ for following wave.

And $\boldsymbol{\mu}$ is 90^0 for beam waves

$$C = \frac{\omega}{k} = \frac{L}{T}$$

Where

V is the vessel speed

C is the wave speed

$\boldsymbol{\mu}$ is the angle of attack

T_e is the encounter period

$$T_e = \frac{\lambda}{C + Vcos(\mu - \pi)} = \frac{\lambda}{C - Vcos\mu} \quad 5$$

$$\omega e = \frac{2\pi}{T_e} = \frac{2\pi(C - Vcos\mu)}{\lambda} = K(C - Vcos\mu) \quad 6$$

But note that;

$$KC = \omega$$

$$\omega e = \omega - \frac{\omega^2}{g} Vcos\mu = \omega(1 - \frac{\omega V}{g} cos\mu) \quad 7$$

Where the angle of attack ($\boldsymbol{\mu}$) ranges between 0^0 and 180^0

The structure will respond in head waves than the following waves this has been base on the assumption that the vessels considered here, is anchored at the aft (turret). The following response will occur on the six degree of freedom as presented in equation 8 to 16 having neglected roll, pitch and yaw in this analysis due to negligible.

Surge displacement = $X = Xacos(\omega et + \mathcal{E}x)$ 8

Surge velocity $= V = -\omega e\ XaSin(\omega et + \mathcal{E}x)$
$= V = \omega e\ XaCos(\omega et + \mathcal{E}x + \frac{\pi}{2})$ 9

Surge acceleration
$a = -\omega e^2\ XaCos(\omega et + \mathcal{E}x)$
$a = \omega e^2\ XaCos(\omega et + \mathcal{E}x + \pi)$ 10

Sway displacement = $Y = Yacos(\omega et + \mathcal{E}y)$ 11

Sway velocity = $V = -\omega e\ YaSin(\omega et + \mathcal{E}x)$
$V = \omega e\ YaCos(\omega et + \mathcal{E}x + \frac{\pi}{2})$ 12

Sway acceleration $a = -\omega e^2\ YaCos(\omega et + \mathcal{E}x)$
$a = \omega e^2\ YaCos(\omega et + \mathcal{E}x + \pi)$ 13

Heave Displacement $Z = Zacos(\omega et + \mathcal{E}z)$ 14

Heave velocity $V = -\omega e\ ZaSin(\omega et + \mathcal{E}x)$
$V = \omega e\ ZaCos(\omega et + \mathcal{E}x + \frac{\pi}{2})$ 15

Heave acceleration $(a) = -\omega e^2\ ZaCos(\omega et + \mathcal{E}x)$
$a = \omega e^2\ ZaCos(\omega et + \mathcal{E}x + \pi)$ 16

Response
= response amplitude × *Cos* (ω_et + phase gap)

$T_e = \frac{\lambda}{C + vcos(\mu - \pi)}$ and $\lambda = \frac{\omega}{\omega z}$

$K = \frac{t}{Za\omega z}$ and $\omega_z = \sqrt{\frac{c}{a}}$

Response amplitude (Z_a) = $Z_{st}\boldsymbol{\mu}$

$Z_{st} = \frac{Fa}{C}$ and phase gap = $\frac{zk\mu}{1-\mu^2}$

Wave amplitude (Z) = $\frac{H}{2}$

$$\mu = \frac{1}{\sqrt{(1-\mu^2)^2 + 4h^2\lambda^2}}$$

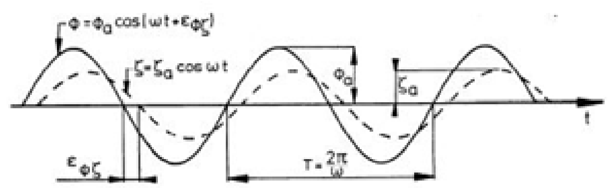

Figure 2: Harmonic wave and surge signal

2.4 Hydrodynamic Theory

The theory which forms the basis of computations of the mean and low frequency second order drift forces (mean and low frequency) on floating structures. This theory is developed based on the assumption that the fluid surrounding the body is in-viscid, irrotational, homogeneous and incompressible.

The fluid motion may be described by a velocity potential

$$\emptyset = \sum_{i=i}^{n} \epsilon i \emptyset i$$

Where ϵi is a small parameter (perturbation) and $\emptyset i$ is the ith order velocity potential such the φ_2 denotes second order velocity potential.

2.5 Coordinate System

The three co-ordinate system of axes is use as presented in figure 1 The first is a right-handed system of G-X_1-X_2-X_3 body axes with as origin the center of gravity G and with positive G- X_3 axis vertically upwards in the mean position of the oscillating vessel. The surface of the hull is uniquely defined relative to this system of axis. A point on the surface has as position the vector x. the orientation of a surface element in this system of axes is defined by the outward pointing normal vector $\vec{n}$

The second system of co-ordinate axes is a fixed O-X_1-X_2-X_3 system with axes parallel to the G-X_1-X_2-X_3 system of axes with the body in the mean position and origin O in the mean free surface.

The third system of co-ordinate axes is a G-X'_1-X'_2-X'_3 system of axes with origin in the center of gravity G of the body and axes which are at all times parallel to the axes of the fixed O-X_1-X_2-X_3 system.

Considering a fixed coordinate system, the pressure at a point on the hull of the FPSO can be determined by writing down the Bernoulli's equation as:

$$p = po - \rho g z - \frac{\partial \phi}{\partial t} - \frac{1}{2}\rho \mid \nabla\varphi \mid^2 \qquad 17$$

Where:

Po = atmospheric pressure

Z = vertical distance of the point below the mean water surface

C (t) = a function independent of the coordinates

T = time

ρ = mass density of the fluid

The quadratic term in equation above can be extended as

$$-\frac{1}{2}\rho \mid \nabla\varphi \mid^2 = -\frac{1}{2}\rho \mid v1^2 + V2^2 + V3^2 \mid \qquad 18$$

Considering an idealized sea state consisting of two wave components of circular frequency ω_1 and ω_2. An approximation for the x-component of the velocity can be written formally as

$$V_1 = A_1 \cos(\omega_1 t + \epsilon_1) + A_2 \cos(\omega_2 t + \epsilon_2) \qquad 19$$

Extending the first velocity terms of equation 3 for two wave components with different wave amplitude A1 and A2 and of circular frequencies ω_1 and ω_2 propagating in idealized sea state lead to:

$$-\frac{1}{2}\rho V^2 = \frac{\rho}{2}\left[\frac{A1^2}{2} + \frac{A2^2}{2} + \frac{A1^2}{2}\cos(2\omega i t + 2\mathcal{E}i) + \frac{A2^2}{2}\cos(2\omega i t + 2\mathcal{E}2) + A1A2\cos\big((\omega 1 + \omega 2)t + \mathcal{E}1 - \mathcal{E}2\big) + A1A2\cos\big((\omega 1 + \omega 2)t + \mathcal{E}1 - \mathcal{E}2\big)\right] \qquad 20$$

This equation shows that second order effects are generally those effects which are their linear with the wave amplitude or proportional to the square of the wave amplitude. It can be analyzed such that the pressure constant term $-\frac{\rho}{2}\left[\frac{A1^2}{2} + \frac{A2^2}{2}\right]$ represent steady pressure.

3 Result Analysis

The first task in a global response analysis is to identify the static position of the structure that is, an act of establishing a reference point. Thus any deviation from this static position is defined as the response of the structure. The lateral motion include sway, surge and heave motion. These responses have been calculated and tabulated in table 1-6. Some of the parameters used in the lateral response calculations are encounter wave period, encounter frequency, wavelength, phase angle and amplitude. Since the system is linear, the resulting motion in waves can be seen as a superposition of the motion of the body in still water and the forces on the restrained body in waves. Thus, two important assumptions are made here for the loads on the right hand side of the figure 3. The so-called hydro mechanical forces and moments are induced by the harmonic oscillations of the rigid body, moving in the undisturbed surface of the fluid.

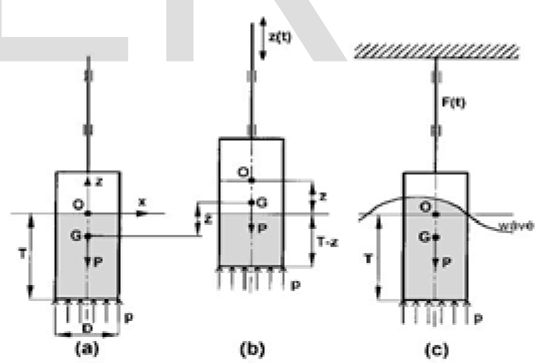

Figure 3: Heaving Circular Cylinder. ()

These frequency characteristics are known, for instance via model experiments or computations. In many cases the FPSO motions have mainly been a linear behaviour. This means that, at each frequency, the ratios between the motion amplitudes and the wave amplitudes and also the phase shifts yields the motions. Doubling the input (wave) amplitude results in a doubled output amplitude making the relationship directly reciprocal, while the phase shifts between output and input does not change. [7][4] As a consequence of the linear theory, the resulting motions in irregular waves can be obtained by adding together results from regular waves of different amplitudes, frequencies and possibly propagation directions. With known wave energy spectra and the calculated frequency characteristics of the responses of the ship shaped structure, the response spectra and the statistics of these responses can be established. [8][5]

Table 1: Surge motion response of an FPSO on following waves

Wave period (s)	Wave frequency (rad/s)	ω^2	Wavelenght (m)	Surge response (m)
8	0.775	0.600625	102.5598	0.05
8.5	0.729412	0.532042	115.7804	0.19
9	0.688889	0.474568	129.8023	0.17
9.3	0.666667	0.444444	138.6	0.21
9.8	0.632653	0.40025	153.9039	0.41
10	0.62000	0.3844	160.2497	0.42
10.6	0.584906	0.342115	180.0566	0.43
11	0.563636	0.317686	193.9022	0.68
11.5	0.53913	0.290662	211.9303	0.7
11.8	0.525424	0.27607	223.1317	0.64
12	0.516667	0.266944	230.7596	0.7
12.4	0.5	0.25	246.4	0.79
12.7	0.488189	0.238328	258.4668	0.93
13	0.476923	0.227456	270.8221	0.81

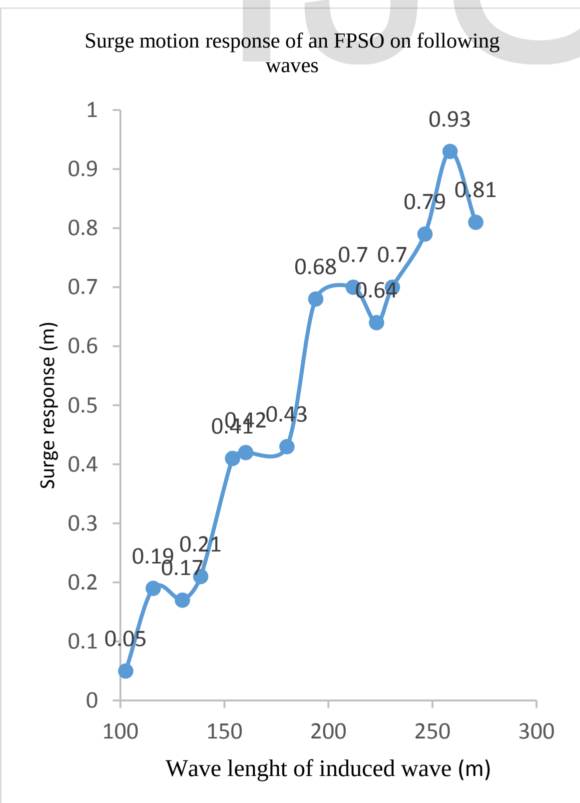

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Figure 4: Surge motion response of an FPSO on following waves.

Table 2: Surge motion response of an FPSO at 90^0 (Beam waves)

T (s)	f	ω^2	L (m)	Surge response (m)
8	0.775	0.600625	102.5598	0.1
8.5	0.729412	0.532042	115.7804	0.38
9	0.688889	0.474568	129.8023	0.34
9.3	0.666667	0.444444	138.6	0.42
9.8	0.632653	0.40025	153.9039	0.82
10	0.62	0.3844	160.2497	0.84
10.6	0.584906	0.342115	180.0566	0.86
11	0.563636	0.317686	193.9022	1.36
11.5	0.53913	0.290662	211.9303	1.4
11.8	0.525424	0.27607	223.1317	1.28
12	0.516667	0.266944	230.7596	1.4
12.4	0.5	0.25	246.4	1.58
12.7	0.488189	0.238328	258.4668	1.86
13	0.476923	0.227456	270.8221	1.62

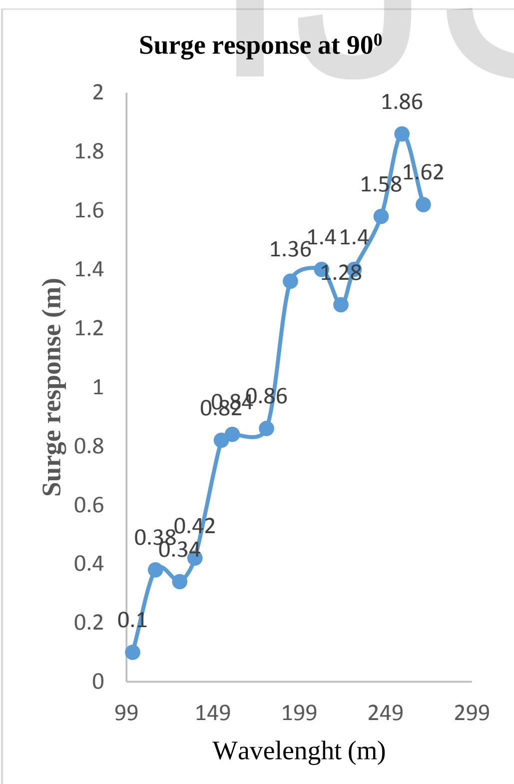

Figure 5: Surge motion response of an FPSO on beam waves.

Table 3: Sway response (m) at 0^0 (following waves)

Period	frequency	ω^2	Wavelength (m)	sway response (m)
7	0.885714	0.78449	78.52237	0.01
7.6	0.815789	0.665512	92.56025	0.1
8.3	0.746988	0.557991	110.396	0.1
8.7	0.712644	0.507861	121.293	0.2
9.1	0.681319	0.464195	132.7028	0.35
10	0.62	0.3844	160.2497	0.25
10.6	0.584906	0.342115	180.0566	0.28
11	0.563636	0.317686	193.9022	0.26
11.4	0.54386	0.295783	208.2606	0.27
11.9	0.521008	0.27145	226.9297	0.5
12	0.516667	0.266944	230.7596	0.9
12.3	0.504065	0.254082	242.4418	1.2
12.6	0.492063	0.242126	254.4125	1.8
12.9	0.48062	0.230996	266.6716	2.4

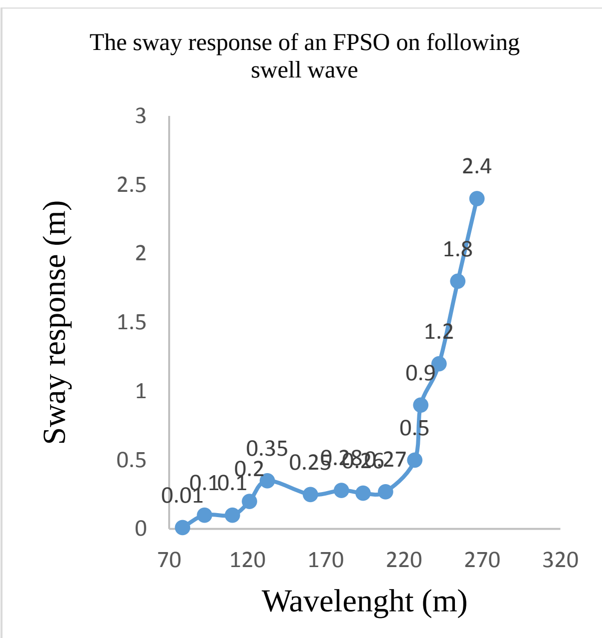

Figure 6: sway motion response at 0^0 angle of attack.

From the above figure (Fig. 6) the plot interpretation is that at a wave length of 120m the sway response at 0^0 attack angle, the sway response will be 0.2m.

Table 4: Sway response at 90^0 (Beam waves)

Period (s)	frequency	ω^2	Wavelength (m)	Sway response (m)
7	0.885714	0.78449	78.52237	0.0162
7.6	0.815789	0.665512	92.56025	0.162
8.3	0.746988	0.557991	110.396	0.162
8.7	0.712644	0.507861	121.293	0.324
9.1	0.681319	0.464195	132.7028	0.567
10	0.62	0.3844	160.2497	0.405
10.6	0.584906	0.342115	180.0566	0.4536
11	0.563636	0.317686	193.9022	0.4212
11.4	0.54386	0.295783	208.2606	0.4374
11.9	0.521008	0.27145	226.9297	0.81
12	0.516667	0.266944	230.7596	1.458
12.3	0.504065	0.254082	242.4418	1.944
12.6	0.492063	0.242126	254.4125	2.916
12.9	0.48062	0.230996	266.6716	3.888

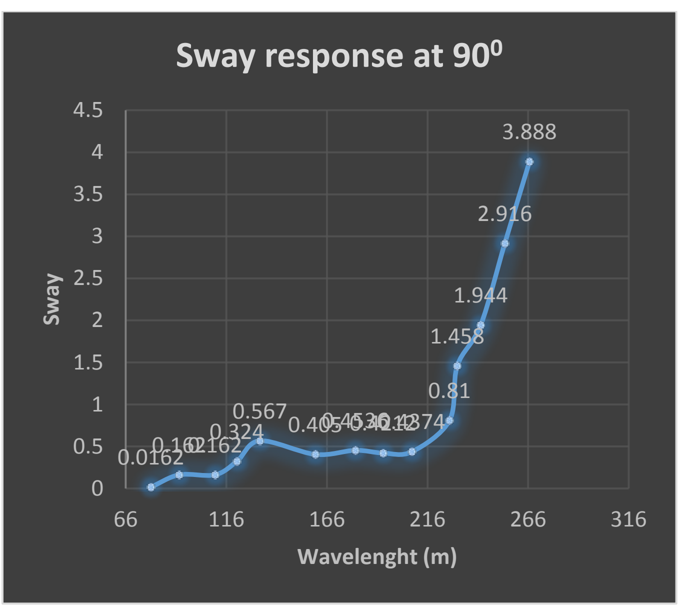

Figure 7: sway motion response at 90^0 angle of attack.

The above plot shows the relationship between the sway displacements and the wave length obtained from encounter frequency and period.

Table 5: Heave response at 0^0 angle of attack.

Period (s)	Frequency (Hz)	ω^2	Wavelength (m)	Heave response (m)
10	0.62	0.3844	160.2497	0
10.4	0.596154	0.355399	173.3261	0.09
10.6	0.584906	0.342115	180.0566	0.15
11.5	0.53913	0.290662	211.9303	0.17
12.45	0.497992	0.247996	248.3911	1.15
13.2	0.469697	0.220615	279.2191	0.21
13.9	0.446043	0.198955	309.6185	0.37
14.4	0.430556	0.185378	332.2939	1.81
14.9	0.416107	0.173145	355.7704	1.41
15	0.413333	0.170844	360.5619	1.52
15.56	0.398458	0.158768	387.9864	1.69
16	0.3875	0.150156	410.2393	1.55
17	0.364706	0.13301	463.1217	1.8
18	0.344444	0.118642	519.2092	2.4

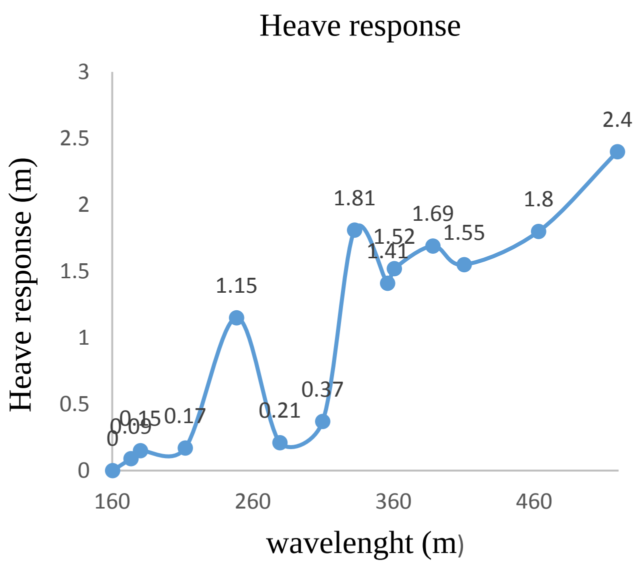

Figure 8: Heave motion response of FPSO at 0^0 angle of attack.

Table 6: Heave response at 90^0 angle of attack

Period (s)	Frequency	ω^2	Wavelength (m)	Heave response (m)
10	0.62	0.3844	160.2497	0
10.4	0.596154	0.355399	173.3261	0.099
10.6	0.584906	0.342115	180.0566	0.165
11.5	0.53913	0.290662	211.9303	0.187
12.45	0.497992	0.247996	248.3911	1.265
13.2	0.469697	0.220615	279.2191	0.231
13.9	0.446043	0.198955	309.6185	0.407
14.4	0.430556	0.185378	332.2939	1.991
14.9	0.416107	0.173145	355.7704	1.551
15	0.413333	0.170844	360.5619	1.672
15.56	0.398458	0.158768	387.9864	1.859
16	0.3875	0.150156	410.2393	1.705
17	0.364706	0.13301	463.1217	1.98
18	0.344444	0.118642	519.2092	2.64

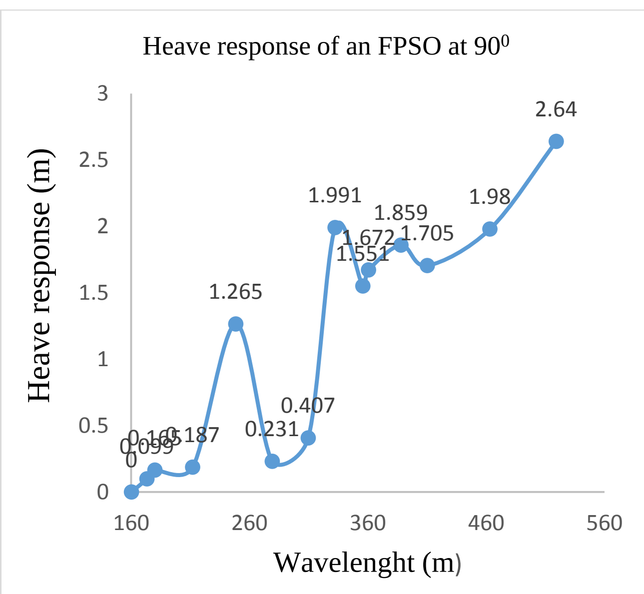

Figure 9: Heave motion response of FPSO at 90^0 angle of attack.

3.1 Reduction of swell wave impact on the structure.

The effect of swell wave on FPSOs would be reduced by employing the principle of added mass, maintaining proper ballasting of tanks during discharge and finally building of swell wave breakers around the structure.

4 Conclusion

From the results obtained in table 1-6, it is obvious that the response of the FPSO is minimal when the wave attack angle is less than 90^0 with the least response obtained at the position of the following angle however it becomes maximum at the heading wave direction. From the response diagram as found in figure 4-9, the degree of response of the structure for surge and sway yields a greater response when the angle of attack is 0^0, whereas the yaw response is greater when the angle of attack is at 90^0 respectively.

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Nomenclature

Symbol	Interpretation
ω	Frequency
k	Wave number
$\boldsymbol{\mu}$	angle of attack
ξ	Wave elevation
φ	Phase angle
Ɛ	Phase gap
λ	Magnification factor
Ů	Velocity potential
σ	Phillip's constant
t	Time
g	Gravity
d	depth
ρ	Density of salt water.
β	Angle between the wave propagation direction and the x-axis.
A	amplitude of the sea spectrum characterizing sea state
ϵj	phase angle which is considered randomly